I B PATEL ENGLISH SCHOOL (SELF-FINANCE)

CLASS – 9

SUBJECT - MATHS

CHAPTER – 1

NUMBER SYSTEMS

Introduction

Let's recall different sets of numbers

What are different types of numbers?

Plotting real numbers on number line

Operations on real numbers

Introduction

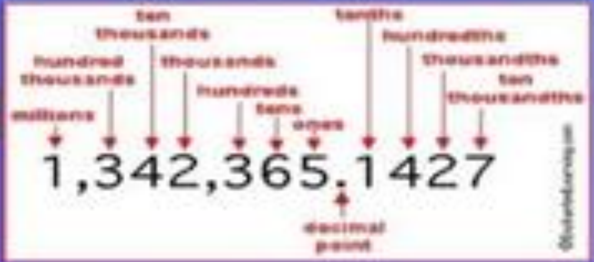
A number system defines a set of values used to represent a quantity. We talk about the number of people attending school, number of modules taken per student etc.

Quantifying items and values in relation to each other is helpful for us to make sense of our environment.

Numbers

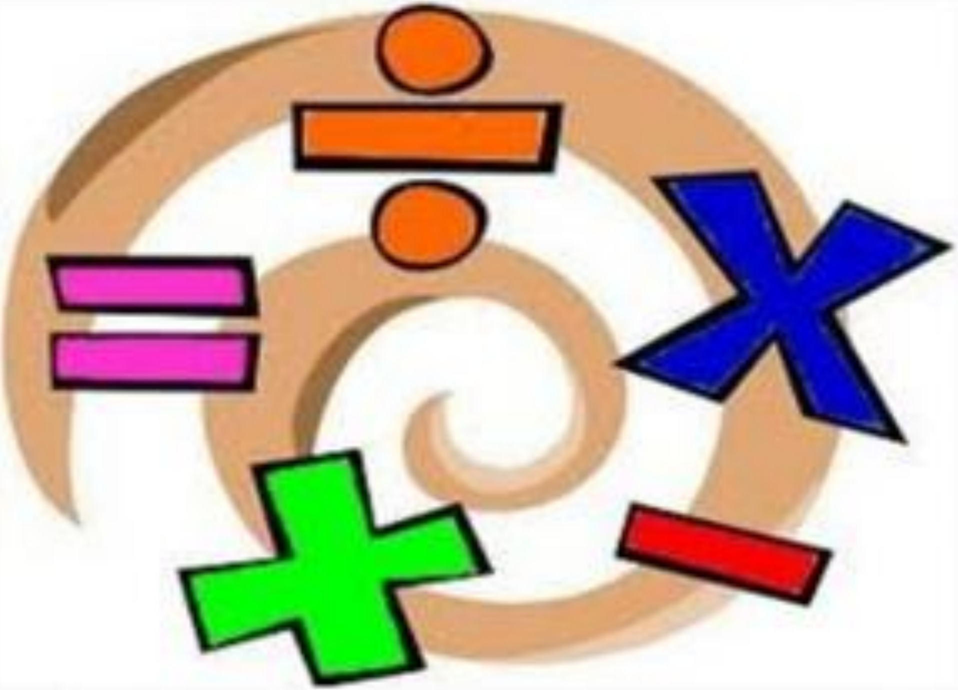
A **number** is a mathematical object used in counting and measuring. It is used in counting and measuring. Numerals are often used for labels, for ordering serial numbers, and for codes like ISBNs. In mathematics, the definition of number has been extended over the years to include such numbers as zero, negative numbers, rational numbers, irrational numbers, and complex numbers.

THE DECIMAL NUMBER SYSTEM



The number system we use on day-to-day basis in the decimal system , which is based on ten digits: zero through nine. As the decimal system is based on ten digits, it is said to be base -10 or radix-10. Outside of specialized requirement such as computing , base-10 numbering system have been adopted almost universally. The decimal system with which we are fated is a place-value system, which means that the value of a particular digit depends both on the itself and on its position within the number.

Types of numbers



Let's recall :

Number System

- Natural numbers N = {1, 2, 3,....}
- Whole numbers W = {0, 1, 2,....}
- Integers Z = { – 2, – 1, 0, 1, 2, 3,.... }
- Rational numbers Q = { $5, -6, 0, \frac{-2}{7}, \frac{-8}{-19}, \frac{5}{6}, \frac{1}{4}, \frac{10}{-3}, ...$ }

COSURE PROPERTY

NUMBERS	CLOSED UNDER			
	ADDITION $a + b$ is a rational number	SUBTRACTION $a - b$ is a rational number	MULTIPLICATION $a \times b$ is a rational number	DIVISION $a \div b$ is a rational number
RATIONAL NUMBERS	YES	YES	YES	YES

COMMUTATIVE PROPERTY

NUMBERS	COMMUTATIVE FOR			
	ADDITION $a + b = b + a$	SUBTRACTION $a - b = b - a$	MULTIPLICATION $a \times b = b \times a$	DIVISION $a \div b = b \div a$
RATIONAL NUMBERS	YES	NO	YES	NO

ASSOCIATIVE PROPERTY

NUMBERS	ASSOCIATIVE FOR			
	ADDITION $(a + b) + c = a + (b + c)$	SUBTRACTION $(a - b) - c = a - (b - c)$	MULTIPLICATION $(a \times b) \times c = a \times (b \times c)$	DIVISION $(a \div b) \div c = a \div (b \div c)$
RATIONAL NUMBERS	YES	NO	YES	NO

Distributive property (Distributive law) for rational numbers :

Distribution of multiplication over addition :

For any three rational numbers a, b and c, $a \times (b + c) = ab + ac$.

$$a(b+c) = ab + ac$$

Natural numbers

- The most familiar numbers are the natural numbers or counting numbers: One, Two, Three and so on….
- Traditionally, the sequence of natural numbers started with 1.However in the 19^{th} century, mathematicians started including 0 in the set of natural numbers.
- The mathematical symbol for the set of all natural numbers is 'N'.

. . . 2, 1, 0, -1, -2, . . .

Integers

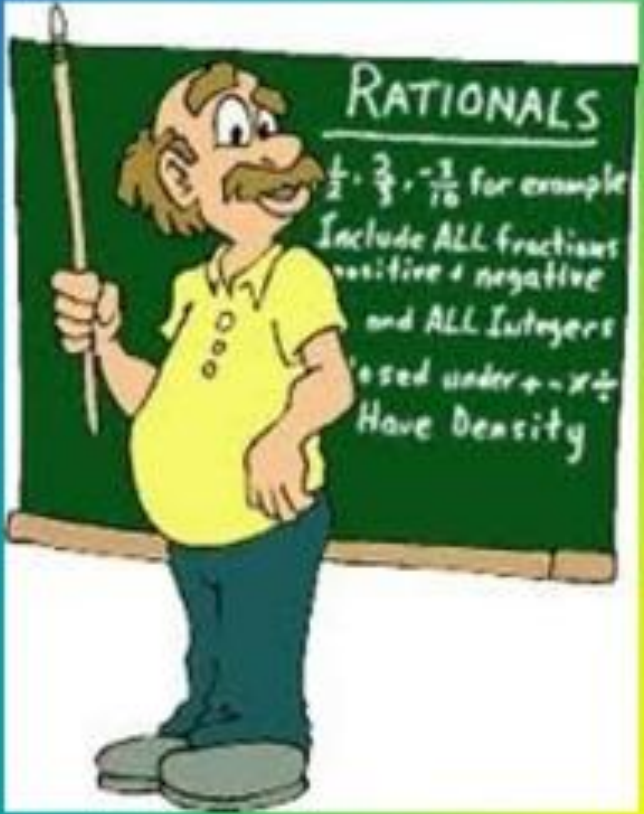
- Integers are the number which includes positive and negative numbers.
- Negative numbers are numbers that are less than zero. They are opposite of positive numbers . Negative numbers are usually written with a negative sign(also called a minus sign)in front of the number they are opposite of .When the set of negative numbers is combined with the natural numbers zero, the result is the set of integer numbers , also called ‘Z’.

Fractions

This is a way of depicting a rational number. Fractions are written as two numbers, the numerator and the denominator ,with a dividing bar between them.

- In the fraction m/n 'm' represents equal parts, where 'n' equal parts of that size make up one whole.
- If the absolute value of m is greater than n ,then the absolute value of the fraction is greater than 1.Fractions can be greater than ,less than ,or equal to1 and can also be positive ,negative , or zero.

Rational Numbers



A rational number is a number that can be expressed as a fraction with an integer numerator and a non-zero natural number denominator. The symbol of the rational number is 'Q'. It includes all types of numbers other than irrational numbers, i.e. it includes integers, whole number, natural numbers etc…

π

Irrational Numbers

If a real number cannot be written as a fraction of two integers, i.e. it is not rational, it is called irrational numbers . A decimal that can be written as a fraction either ends(terminates)or forever repeats about which we will see in detail further.

Real number pi (π) is an example of irrational.
π=3.14159365358979……the number neither start repeating themselves nor come in a specific pattern.

Creamy bonus : Square root of any non-square natural number is an irrational number.

For example,

1, 4, 9, 16, ….. are square numbers and their square roots are 1, 2, 3, 4, ….. (natural numbers).

whereas,

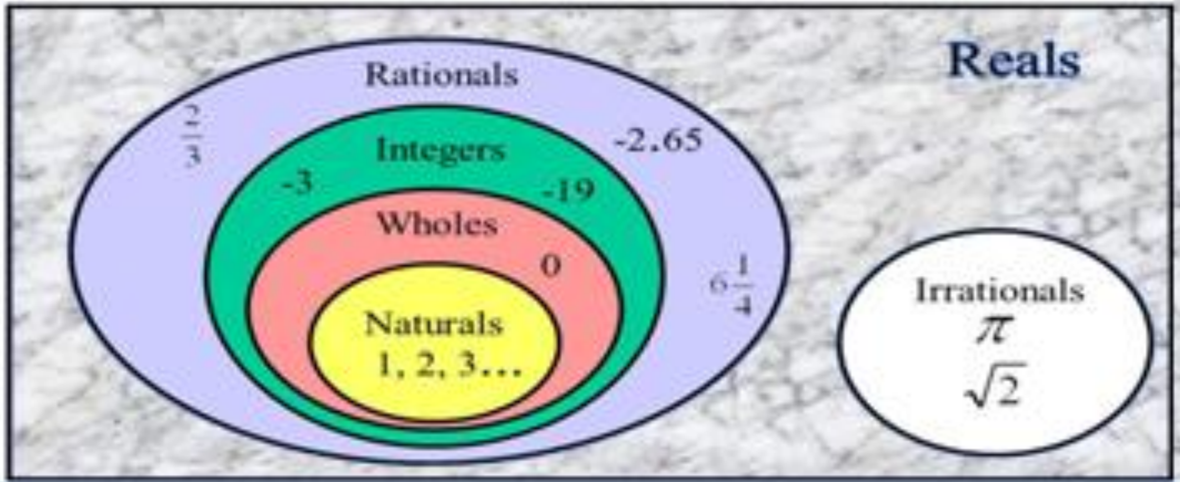
2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, …… are non-square numbers and thus their square roots are irrational numbers.

Real Numbers

The real numbers include all of the measuring numbers . Real numbers are usually written using decimal numerals , in which a decimal point is placed to the right of the digit with place value one.

- It includes all types of numbers such as Integers, Whole numbers, Natural numbers, Rational number, Irrational numbers and etc…

Every real number is represented by a unique point on the number line. Also every point on the number line represents a unique real number.

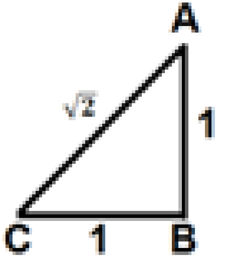


What is a number line?

A number line is a line with marks on it that are placed at equal distance apart. One mark on the number line is usually labeled zero and then each successive mark to the left or to the write of the zero represents a particular unit such as 1, or 0.5. It is a picture of a straight line.

Locating square roots of non-perfect square numbers on number line :

1) By Pythagoras theorem, in a right angled triangle ABC,



$$AC^2 = AB^2 + BC^2$$

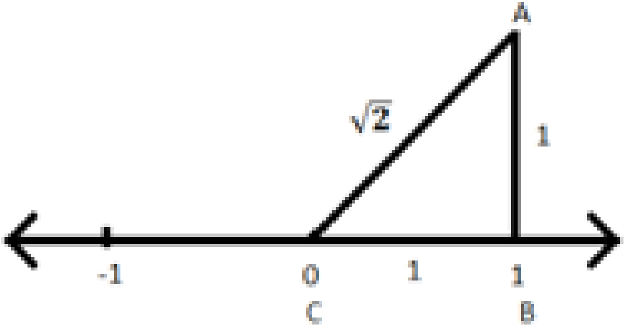
Let us transfer this triangle with each of the two sides measuring one unit on number line, such that the vertex C coincides with 0.

$$AC^2 = AB^2 + BC^2$$

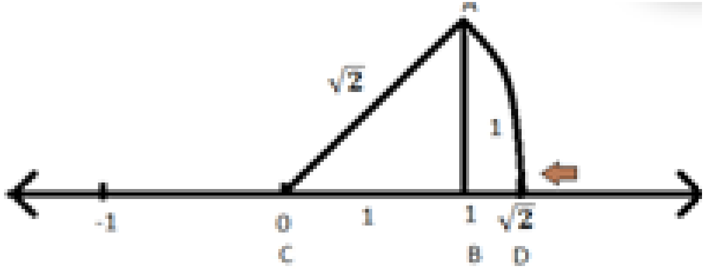
$$\Rightarrow AC^2 = 1^2 + 1^2$$

$$\Rightarrow AC^2 = 2$$

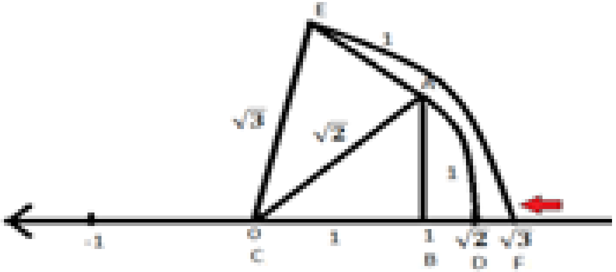
$$\Rightarrow AC = \sqrt{2}$$



As shown in figure, $AC = \sqrt{2}$. Now draw an arc on number line, with centre C and radius AC, which will intersect the line at D. This D point corresponds to $\sqrt{2}$ on number line.



2) Now construct AE of unit length such that it is perpendicular to AC. Using Pythagoras theorem, we have



$$EC^2 = AE^2 + AC^2$$

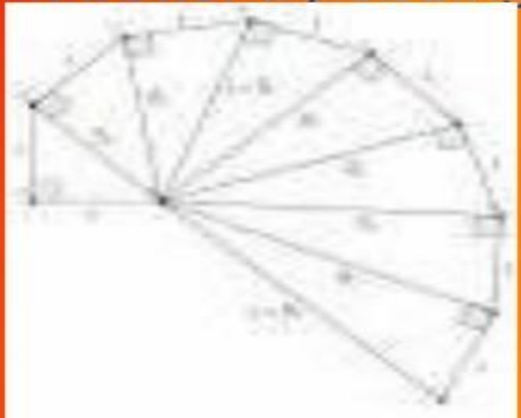
As shown in figure, $EC = \sqrt{3}$. Now draw an arc on number line, with centre C and radius EC, which will intersect the line at F. This F point corresponds to $\sqrt{3}$ on number line.

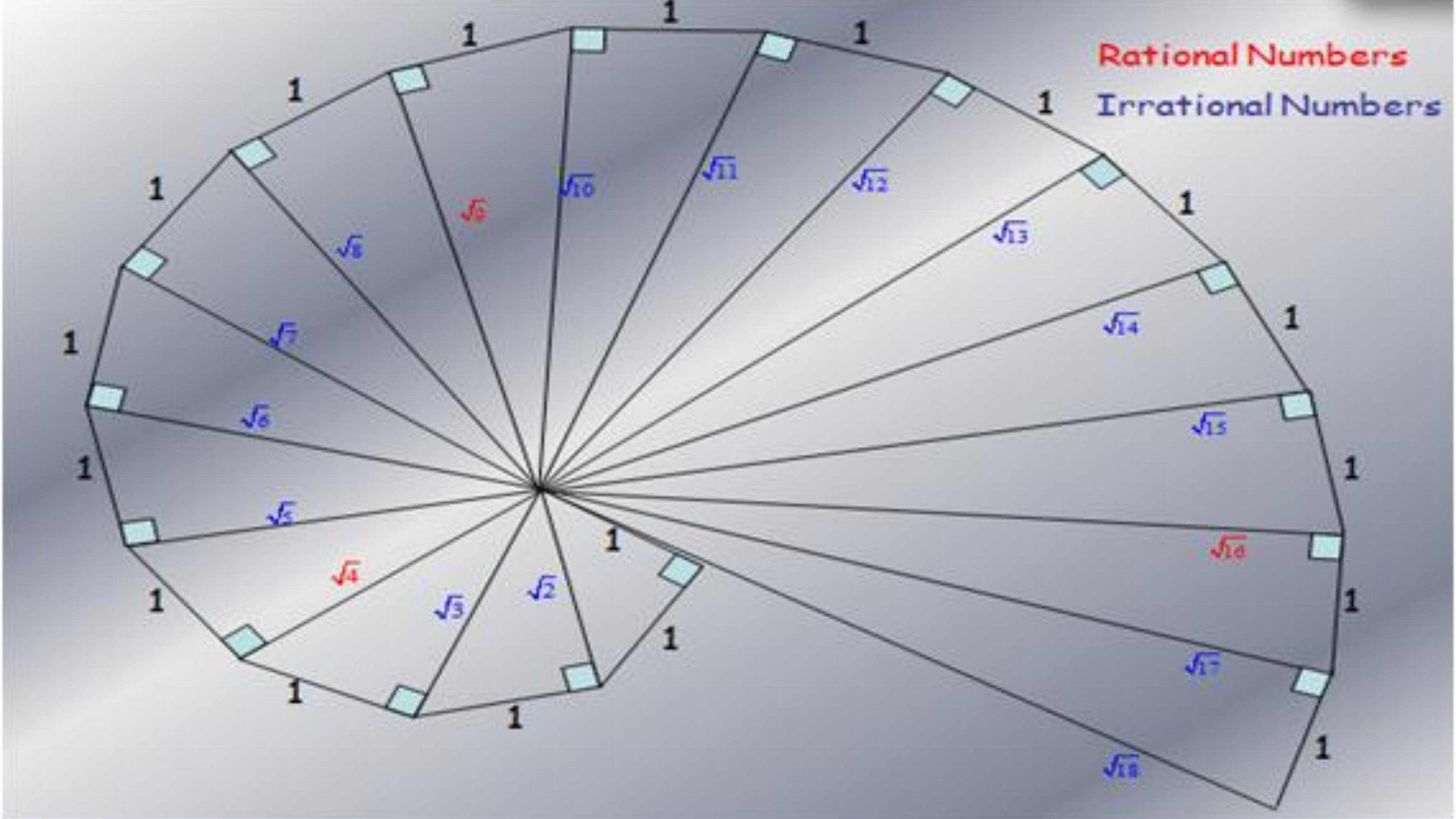
$$EC^2 = AE^2 + AC^2$$
$$\Rightarrow EC^2 = 1^2 + \sqrt{2}^2$$
$$\Rightarrow EC^2 = 1 + 2$$
$$\Rightarrow EC^2 = 3$$
$$\Rightarrow EC = \sqrt{3}$$

Spiral Method to find irrational numbers on the number line

Representation of irrational numbers on number line can be done with the help of Pythagoras theorem. We can generate square roots of numbers using right angle triangles for different irrational numbers. Further, WE can construct a spiral square by representing irrational numbers as diagonal of each square. The pattern generated regarding the length of each square is a geometric pattern.

You will understand it better by seeing this video.

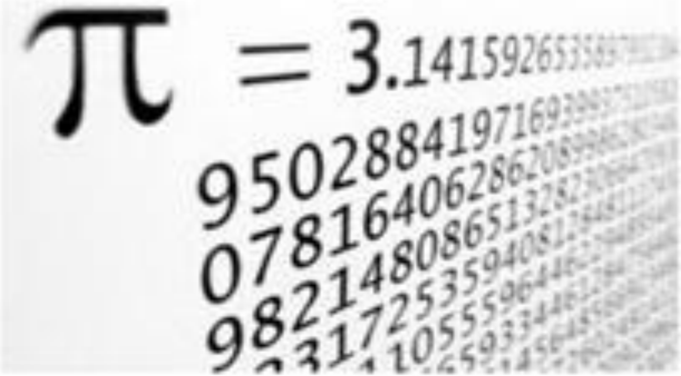



Rational Numbers
Irrational Numbers
1
√2
√3
√4
√5
√6
√7
√8
√10
√11
√12
√13
√14
√15
√16
√17
√18

Decimal Expansion of Numbers

A decimal expansion of a number can be either,

- Terminating
- Non-terminating, non recurring
- Non terminating, recurring



Let us see each of the following briefly…

Terminating decimal

A decimal expansion in which the remainder becomes zero. For example, 54 ÷ 9 =

```
    6
  ____
9 ) 54
    54
  ____
     0
```

As the remainder is zero, this is a terminating decimal

Terminating decimal is always a rational number. It can be written in p/q form.

Show that the terminating decimals below are rational.

$$0.7 = \frac{7}{10}$$

$$0.625 = \frac{5}{8}$$

$$34.56 = \frac{3456}{100}$$

Non terminating, recurring

In this form, when a number is divided by the other, the remainder never becomes zero, instead the numbers of the quotient start repeating themselves. Such numbers are classified as rational numbers. For example,

3.725072507250…

In this example, "7250" have started repeating itself. Hence, it is a rational number. It can be expressed in p/q form.

Show that the repeating decimals below are rational.

Question 1

Show that 0.222… is rational.

Let $x = 0.222\ldots$

$10x = 2.22\ldots$

Multiply both sides by 10

$10x = 2 + 0.22\ldots$

$10x = 2 + x$

$9x = 2$

$x = 2/9$

Question 2

Show that 0.6363… is rational

Let $x = 0.6363\ldots$

$100x = 63.63\ldots$

Multiply both sides by 10

$100x = 63 + 0.63\ldots$

$100x = 63 + x$

$99x = 63$

$x = 63/99$

$x = 7/11$

Crunchy perk :

$$\frac{a}{b}$$

- a ← Repeating block of the decimal number
- b ← Repetition of 9 as many times as the number of digits in numerator

$$0.\overline{3} \rightarrow \frac{3}{9} \qquad 0.\overline{45} \rightarrow \frac{45}{99} \qquad 0.\overline{273} \rightarrow \frac{273}{999} \qquad 0.1\overline{234} \rightarrow \frac{1233}{9990}$$

Non terminating non recurring

3.14159265358979...

"Recurring" means "repeating". In this form, when we divide a number by another, remainder never becomes zero, and also the number does not repeat themselves in any specific pattern. If a number is non terminating and non repeating, they are always classified as irrational number. For example,

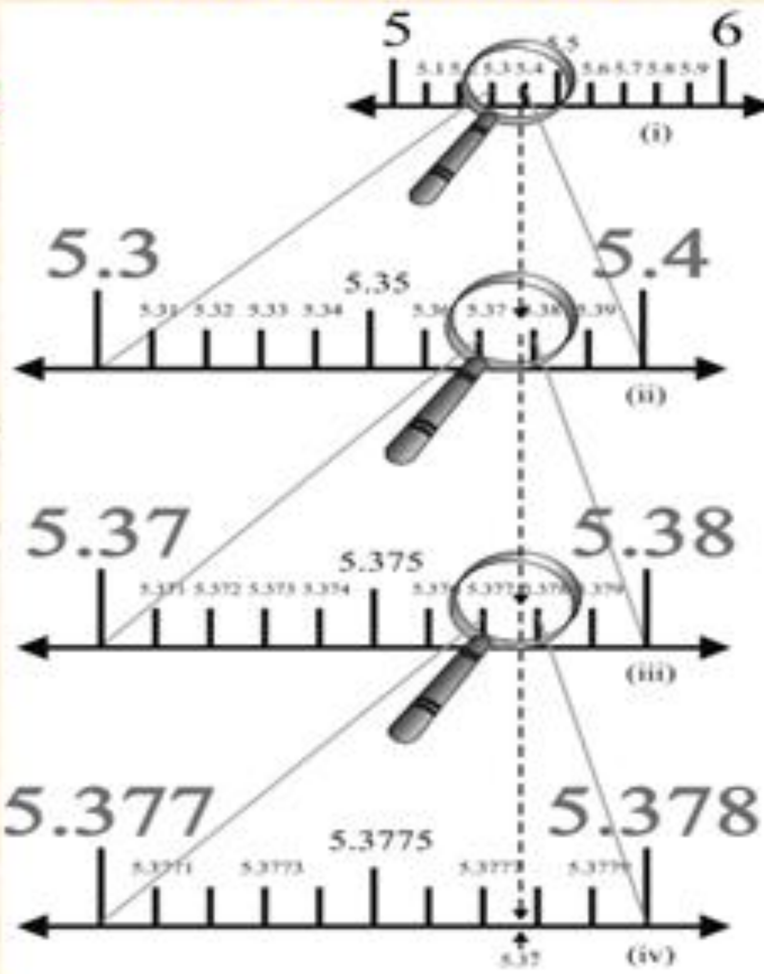
0.101001000100001000001OO.... does have a pattern, but it is not a fixed-length recurring pattern, so the number is irrational.

Representing real numbers on number line :

On the number line, there are infinite smaller numbers lying between any two numbers. These smaller numbers can be of two, three or more decimal places. To see or mark such numbers clearly, we use the process called successive magnification of the number line. Here, we use a virtual (imaginary) magnifying glass to enlarge the smaller divisions on the number line .

This procedure can be used to visualize a real number with a non-terminating non-recurring decimal expansion on the number line.

Finding 5.3775...



Properties of Real Numbers

Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication like rational numbers.

For example, $\left(\sqrt{6}\right)+\left(-\sqrt{6}\right)$ = 1

$\left(\sqrt{2}\right)-\left(\sqrt{2}\right)$ = 1

$\left(\sqrt{3}\right)\cdot\left(\sqrt{3}\right)$ = 1

$\frac{\sqrt{17}}{\sqrt{17}}$ = 1

are rationals.

But they do not satisfy closure property for any of the four operations (addition, subtraction, multiplication and division) like rational numbers.

Operations on Real numbers

RATIONAL + - x ÷ RATIONAL = RATIONAL

RATIONAL + - x ÷ IRRATIONAL = IRRATIONAL

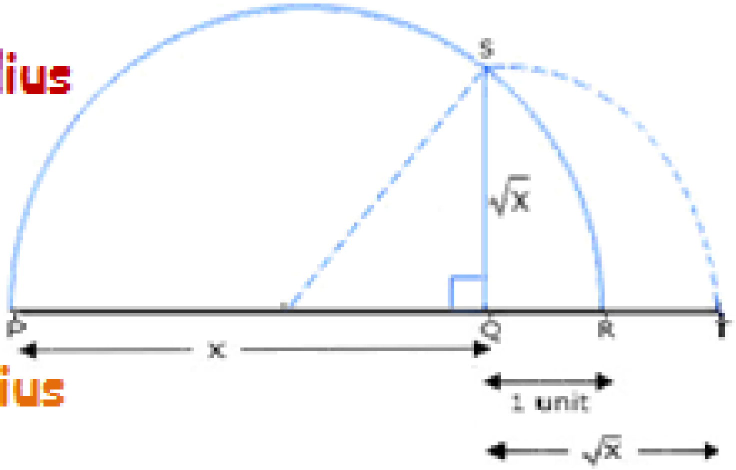
IRRATIONAL + - x ÷ IRRATIONAL = DEPENDS!!!

USUALLY IRRATIONAL,
SOMETIMES THEY SIMPLIFY TO A RATIONAL

Square-root of a positive real number x

Steps :

1) Draw a line-segment PQ of x cm.
2) Extend PQ to point R such that QR = 1 cm.
3) Find out mid-point of PR , say point O.
4) Draw a semi-circle with centre O and radius OR.
5) Draw a perpendicular line from Q on PR which will intersect semi-circle at S.
6) Draw an arc on PR with centre Q and radius QS which will intersect PR at T.
7) QT is $\sqrt{x}$




THANK
YOU