I B PATEL ENGLISH SCHOOL
(PRIMARY SECTION)

CLASS – 8

SUBJECT - MATHS

CHAPTER – 1

RATIONAL NUMBERS

Chapter 1
Rational Numbers

Let's recall different sets of numbers

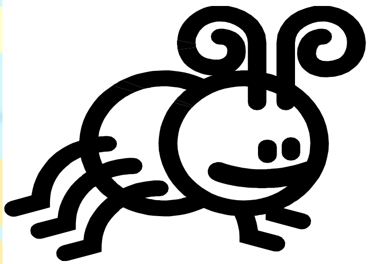
What are rational numbers?

Properties of rational numbers

Plotting rational numbers on number line

Rational numbers between given 2 rational numbers

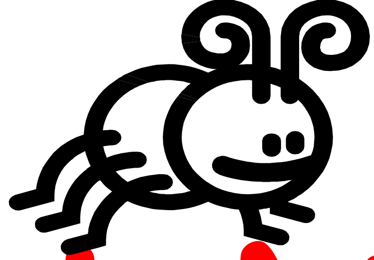
A quick summary



Let's recall :

Number System

- Natural numbers N = {1, 2, 3,…. }
- Whole numbers W = {0, 1, 2,…. }
- Integers Z = { – 2, – 1, 0, 1, 2, 3,…. }



Rational Numbers

1/2

-11/6

0

0.54

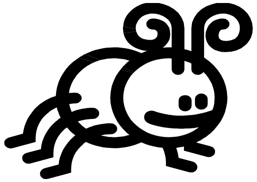
8%

-1.7

-9

3

Rational number - It is a number which can be written in the $\frac{p}{q}$ form where p and q are integers and $q \neq 0$.



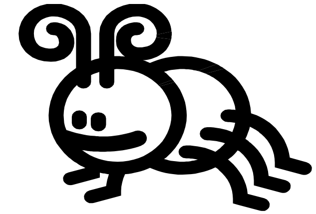
Note: If q is equal to 0 then $\frac{p}{q}$ becomes undefined. So q cannot be equal to 0.

- All the integers are rational numbers.
- Rational numbers are represented by **'Q'**.

Examples -

$5, -6, 0, \frac{-2}{7}, \frac{-8}{-19}, \frac{5}{6}, \frac{1}{4}, \frac{10}{-3}$

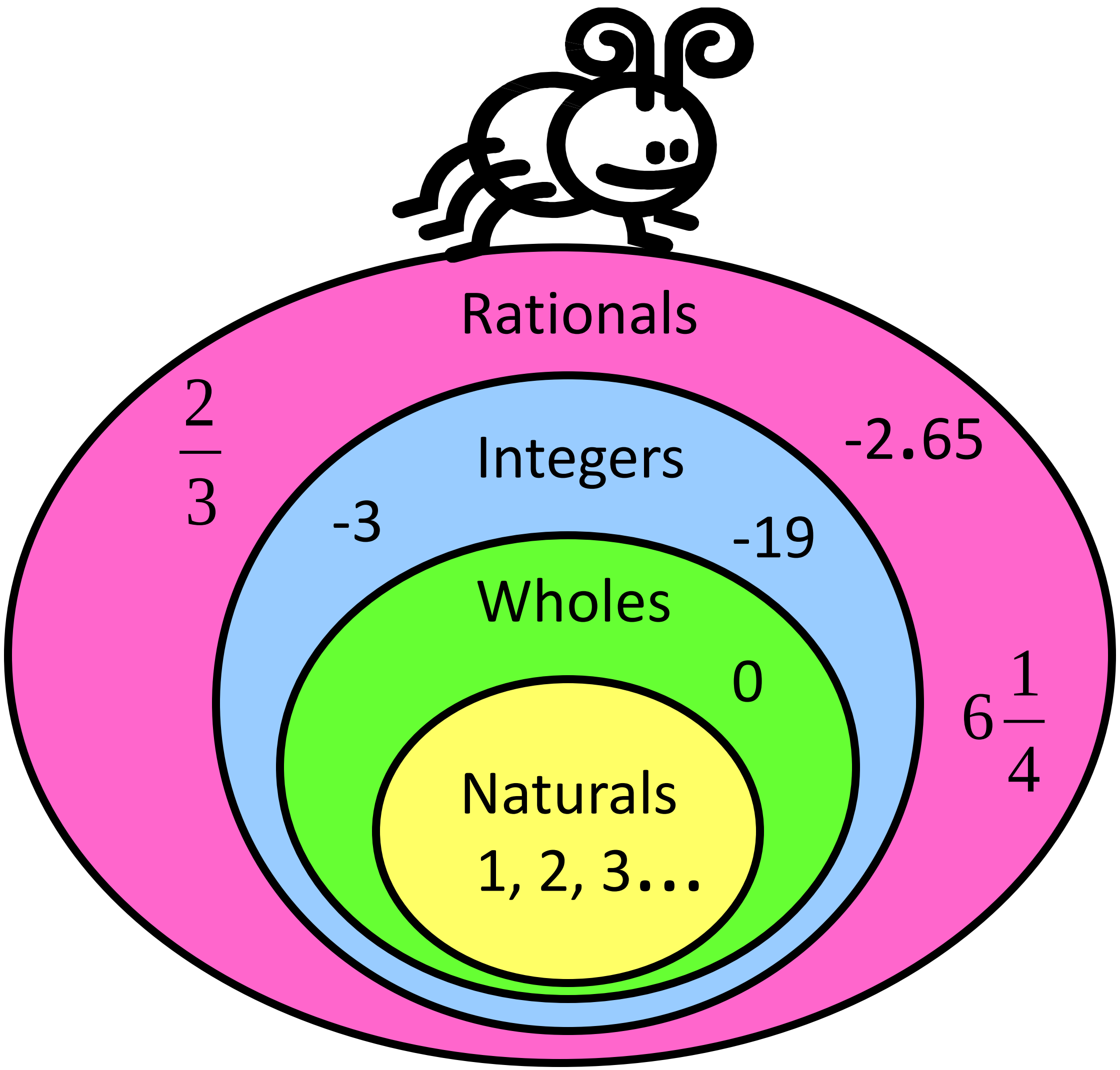
Types of rational numbers



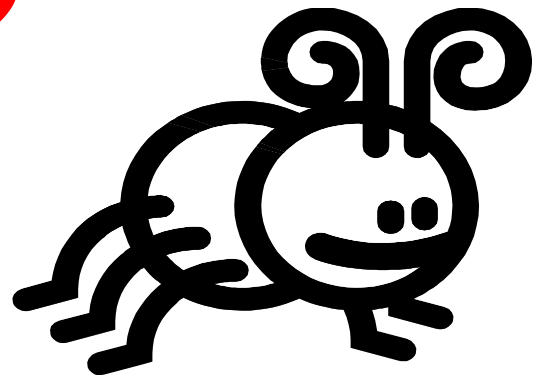
❖ Positive rational numbers = $\frac{-8}{-19}, \frac{5}{6}, \frac{1}{4}$

❖ Negative rational numbers = $\frac{-2}{7}, \frac{10}{-3}$

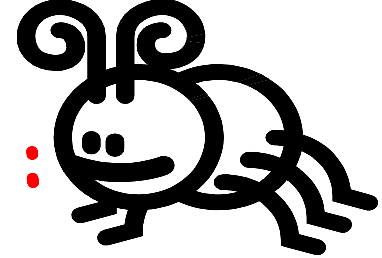
Diagrammatic representation of number sets



Properties of the Rational Numbers



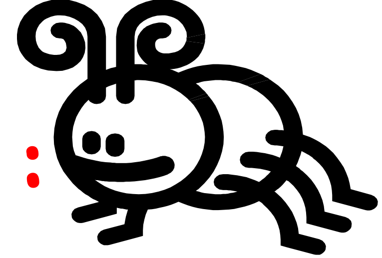
Closure Property of Rational Numbers for :



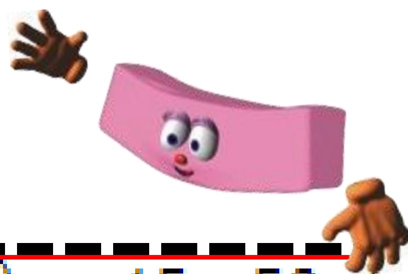
1) **Addition :** Rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number.

$$\frac{4}{7}+\frac{6}{11}=\frac{44+42}{77}=\frac{86}{77}$$

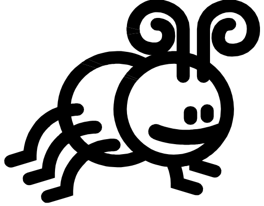
Closure Property of Rational Numbers for :



2) **Subtraction :** Rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number.



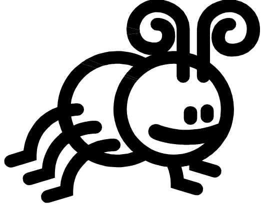
$$\frac{3}{7}-\frac{(-8)}{5}=\frac{15+56}{35}=\frac{71}{35}$$



Closure Property of Rational Numbers for :

3) <u>Multiplication :</u> Rational numbers are closed under multiplication. That is, for any two rational numbers a and b, $a \times b$ is also a rational number.

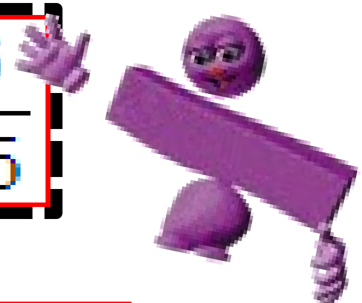
$$\frac{-4}{5} \times \frac{(-6)}{11} = \frac{4 \times 6}{5 \times 11} = \frac{24}{55}$$



Closure Property of Rational Numbers for :

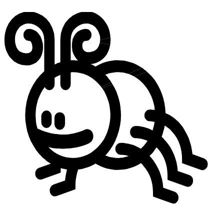
4) <u>Division :</u> Rational numbers are closed under division. That is, for any two rational numbers a and b, $a \div b$ may or may not be a a rational number.

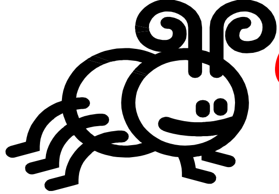
$$\frac{2}{7} \div \frac{5}{3} = \frac{2\times3}{7\times5} = \frac{6}{35}$$



Note : For any rational number a, $a \div 0$ is not defined.

$\frac{6}{0}$ → Undefined



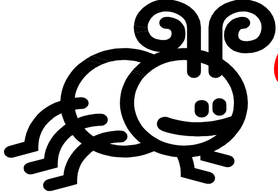


Commutative Property of Rational Numbers for :

1) **Addition :** Rational numbers can be added in any order. So addition is commutative for rational numbers. That is, for any two rational numbers a and b,

$$a + b = b + a$$

L. H. S.	R. H. S.
$\left(\frac{-6}{5}\right) + \left(\frac{-8}{3}\right)$	$\left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right)$
$= \left(\frac{-18 + -40}{15}\right)$	$= \left(\frac{-40 + -18}{15}\right)$
$= \left(\frac{-58}{15}\right)$	$= \left(\frac{-58}{15}\right)$

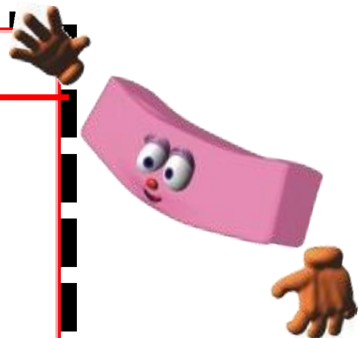


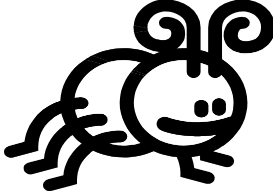
Commutative Property of Rational Numbers for :

2) <u>Subtraction :</u> Subtraction is not commutative for rational numbers. That is, for any two rational numbers a and b,

$$a - b \neq b - a$$

L. H. S.	R. H. S.
$\frac{1}{2} - \frac{3}{5}$	$\frac{3}{5} - \frac{1}{2}$
$= \frac{5 - 6}{10}$	$= \frac{6 - 5}{10}$
$= \frac{-1}{10}$	$= \frac{1}{10}$



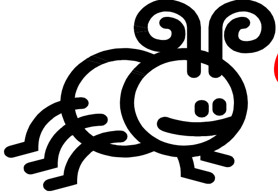


Commutative Property of Rational Numbers for :

3) **<u>Multiplication :</u>** Rational numbers can be multiplied in any order. So multiplication is commutative for rational numbers. That is, for any two rational numbers a and b,

$$a \times b = b \times a$$

L. H. S.	R. H. S.
$\left(\frac{-8}{9}\right) \times \left(\frac{-4}{7}\right)$	$\left(\frac{-4}{7}\right) \times \left(\frac{-8}{9}\right)$
$= \left(\frac{-8 \times -4}{9 \times 7}\right)$	$= \left(\frac{-4 \times -8}{7 \times 9}\right)$
$= \left(\frac{32}{63}\right)$	$= \left(\frac{32}{63}\right)$

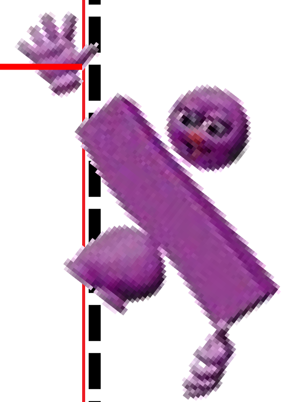


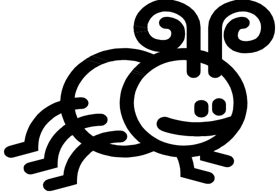
Commutative Property of Rational Numbers for :

4) <u>Division :</u> Division is not commutative for rational numbers. That is, for any two rational numbers a and b,

$$a \div b \neq b \div a$$

L. H. S.	R. H. S.
$\left(\frac{-5}{4}\right) \div \frac{3}{7}$	$\frac{3}{7} \div \left(\frac{-5}{4}\right)$
$= \left(\frac{-5}{4}\right) \times \frac{7}{3}$	$= \frac{3}{7} \times \left(\frac{4}{-5}\right)$
$= \left(\frac{-35}{12}\right)$	$= \left(\frac{12}{-35}\right)$



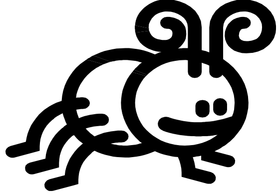


Associative Property of Rational Numbers for :

1) **Addition :** Addition is associative for rational numbers. That is, for any three rational numbers a, b and c,

$$a + (b + c) = (a + b) + c$$

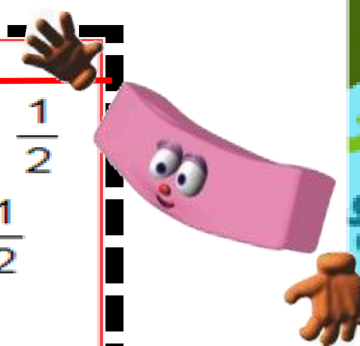
L. H. S.	R. H. S.
$\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]$	$\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)$
$=\frac{-2}{3}+\left(\frac{-7}{30}\right)$	$=\frac{-1}{15}+\left(\frac{-5}{6}\right)$
$=\frac{-27}{30}$	$=\frac{-27}{30}$
$=\frac{-9}{10}$	$=\frac{-9}{10}$



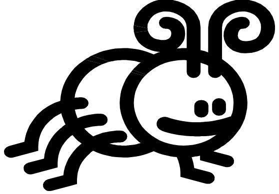
Associative Property of Rational Numbers for :

2) Subtraction : Subtraction is not associative for rational numbers. That is, for any three rational numbers a, b and c,

$$a - (b - c) = (a - b) - c$$



L. H. S.	R. H. S.
$\frac{-2}{3} - \left[\frac{-4}{5} - \frac{1}{2}\right]$	$\left[\frac{-2}{3} - \frac{-4}{5}\right] - \frac{1}{2}$
$= \frac{-2}{3} - \left[\frac{-8-5}{10}\right]$	$= \left[\frac{-10+12}{15}\right] - \frac{1}{2}$
$= \frac{-2}{3} - \left[\frac{-13}{10}\right]$	$= \left[\frac{2}{15}\right] - \frac{1}{2}$
$= \frac{-2}{3} + \frac{13}{10}$	$= \frac{2}{15} - \frac{1}{2}$
$= \frac{-20+39}{30}$	$= \frac{4-15}{30}$
$= \frac{13}{30}$	$= \frac{-11}{30}$

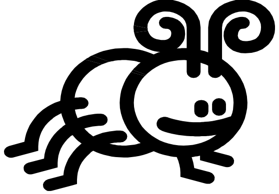


Associative Property of Rational Numbers for :

3) **Multiplication :** Multiplication is associative for rational numbers. That is, for any three rational numbers a, b and c,

$$a \times (b \times c) = (a \times b) \times c$$

L. H. S.	R. H. S.
$\frac{2}{3} \times \left[\frac{-6}{7} \times \frac{4}{5}\right]$	$\left[\frac{2}{3} \times \frac{-6}{7}\right] \times \frac{4}{5}$
$= \frac{2}{3} \times \left[\frac{-24}{35}\right]$	$= \left[\frac{-12}{21}\right] \times \frac{4}{5}$
$= \frac{-48}{105}$	$= \frac{-48}{105}$

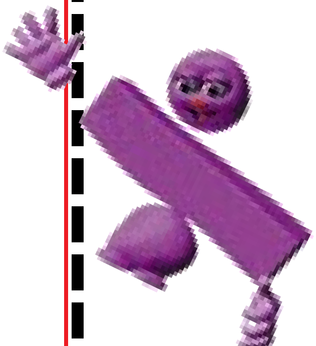


Associative Property of Rational Numbers for :

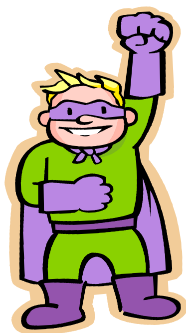
4) <u>Division :</u> Division is not associative for rational numbers. That is, for any three rational numbers a, b and c,

$$a \div (b \div c) \neq (a \div b) \div c$$

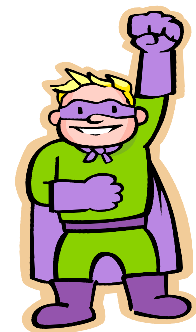
L. H. S.	R. H. S.
$\frac{1}{2} \div \left(\frac{-1}{3} \div \frac{2}{5}\right)$	$\left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$
$= \frac{1}{2} \div \left(\frac{-1}{3} \times \frac{5}{2}\right)$	$= \left(\frac{1}{2} \times \frac{-3}{1}\right) \div \frac{2}{5}$
$= \frac{1}{2} \div \left(-\frac{5}{6}\right)$	$= \frac{-3}{2} \div \frac{2}{5}$
$= \frac{1}{2} \times \left(-\frac{6}{5}\right)$	$= \frac{-3}{2} \times \frac{5}{2}$
$= \left(-\frac{3}{5}\right)$	$= \left(-\frac{15}{4}\right)$



Additive identity (Role of 0): Addition of 0 to a rational number gives back same rational number. That is, for any rational number a, a + 0 = a. So 0 is called the additive identity for rational numbers.

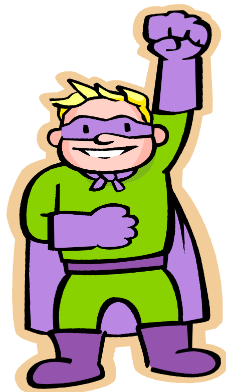


+ 0 =

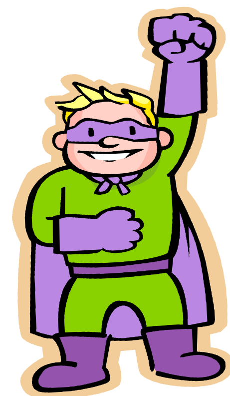


$$\frac{-2}{7} + 0 = \frac{-2}{7}$$

Multiplicative identity (Role of 1): Multiplication of 1 to a rational number gives back same rational number. That is, for any rational number a, $a \times 1 = a$. So 1 is called the multiplicative identity for rational numbers.



× 1 =



$$\frac{8}{15} \times 1 = \frac{8}{15}$$

Additive inverse (Negative of a number): If sum of any two rational numbers is 0, then the two numbers are said to be additive inverse (negative numbers) of each other.

$$\frac{9}{7}+\left(\frac{-9}{7}\right)=\frac{9+(-9)}{7}=\frac{0}{7}=0$$

So $\frac{9}{7}$ is additive inverse of $\left(\frac{-9}{7}\right)$ and vice versa.

Multiplicative inverse (Reciprocal): If product of any two rational numbers is 1, then the two numbers are said to be multiplicative inverse (reciprocal) of each other.

$$\frac{-3}{5} \times \frac{5}{-3} = 1$$

So $\frac{-3}{5}$ is reciprocal of $\frac{5}{-3}$ and vice versa.

Distributive property (Distributive law) for rational numbers :

Distribution of multiplication over addition :

For any three rational numbers a, b and c, $a \times (b + c) = ab + ac$.

Consider the rational numbers, $\frac{-1}{2}$, $\frac{2}{3}$ and $\frac{-3}{5}$

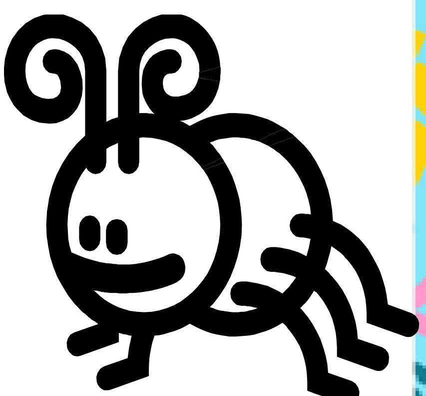
$$a \times (b + c) = a \times b + a \times c$$

$$\frac{-1}{2} \times \left(\frac{2}{3} + \frac{(-3)}{5}\right) = \left(\frac{-1}{2} \times \frac{2}{3}\right) + \left(\frac{-1}{2} \times \frac{-3}{5}\right)$$

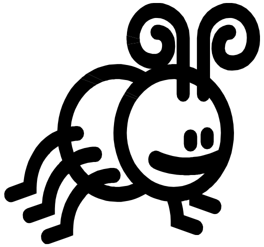
$$\Rightarrow \frac{-1}{2} \times \left(\frac{10-9}{15}\right) = \frac{-2}{6} + \frac{3}{10}$$

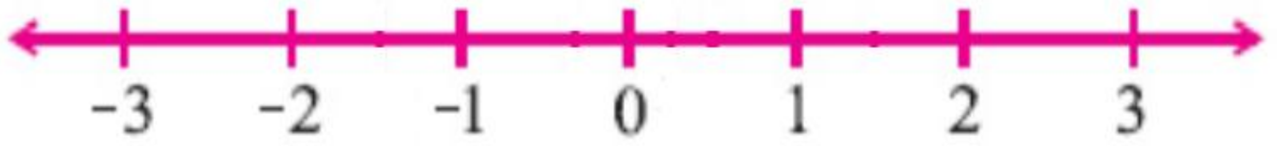
$$\Rightarrow \frac{-1}{2} \times \left(\frac{1}{15}\right) = \frac{-10+9}{30}$$

$$\Rightarrow \frac{-1}{30} = \frac{-1}{30}$$

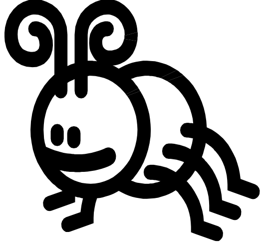


Representation of Rational Numbers on the Number Line





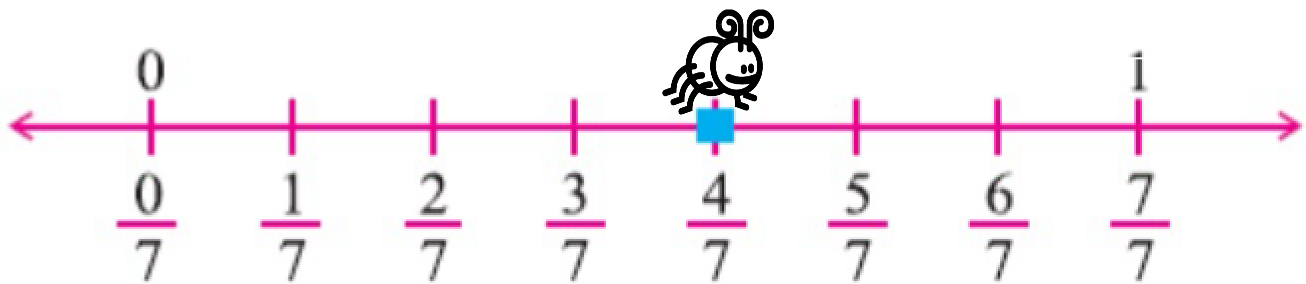
To express rational numbers appropriately on the number line, divide each unit length into as many number of equal parts as the denominator of the rational number and then mark the given number on the number line.

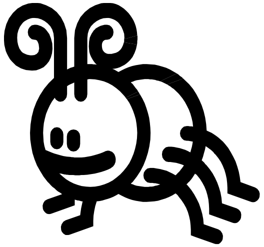


1) Express $\frac{4}{7}$ on the number line.

$\frac{4}{7}$ lies between 0 and 1, and it's denomiator is 7.

So divide the space between 0 and 1 on number line into 7 equal parts and label the parts on scale as shown in figure. Mark $\frac{4}{7}$ on the number line.



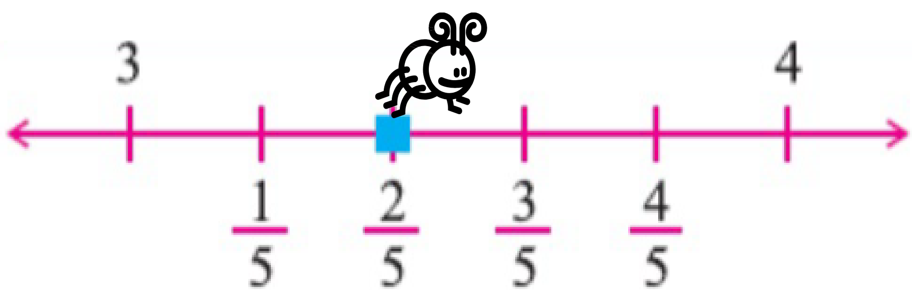


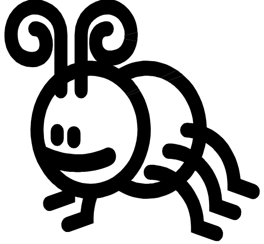
2) Express $\frac{17}{5}$ on the number line.

$\frac{17}{5} = 3\frac{2}{5}$

$3\frac{2}{5}$ lies between 3 and 4, and it's denomiator is 5.

So divide the space between 3 and 4 on number line into 5 equal parts and label the parts on scale as shown in figure. Mark $\frac{2}{5}$ after 3 on the number line.

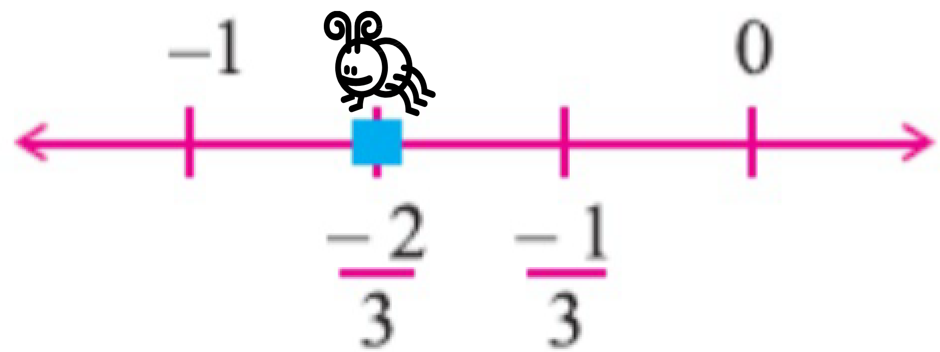




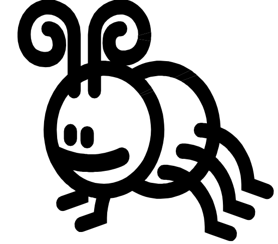
3) Express $\frac{-2}{3}$ on the number line.

$\frac{-2}{3}$ lies between -1 and 0, and it's denomiator is 3.

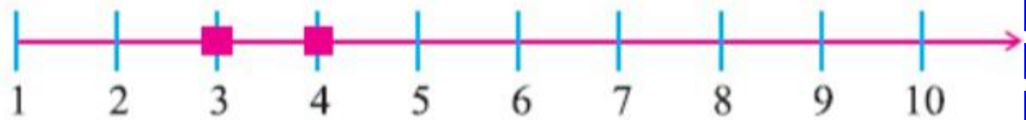
So divide the space between -1 and 0 on number line into 3 equal parts and label the parts on scale as shown in figure. Mark $\frac{-2}{3}$ on the number line.



Rational numbers between two rational numbers :

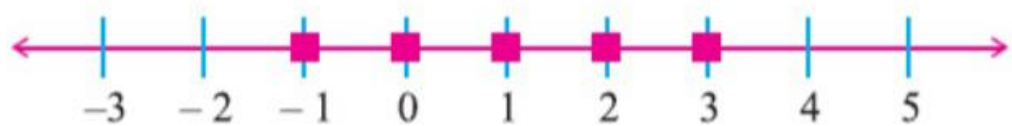


Can you tell the natural numbers between 2 and 5?



They are 3 and 4.

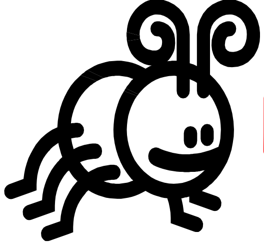
Can you tell the integers between – 2 and 4?



They are – 1, 0, 1, 2, 3.

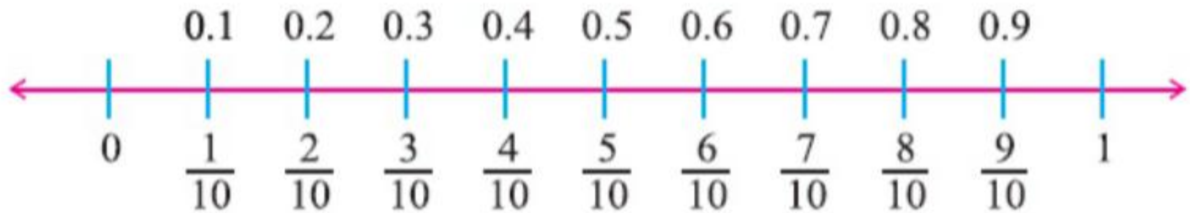
Now, Can you find any integer between 1 and 2?

No.

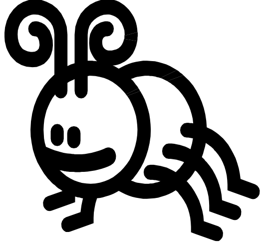


Rational numbers between two rational numbers :

But, between any two integers, we have rational numbers.For example, between 0 and 1, we can find rational numbers $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \cdots$ which can be written as 0.1, 0.2, 0.3, $\cdots$.

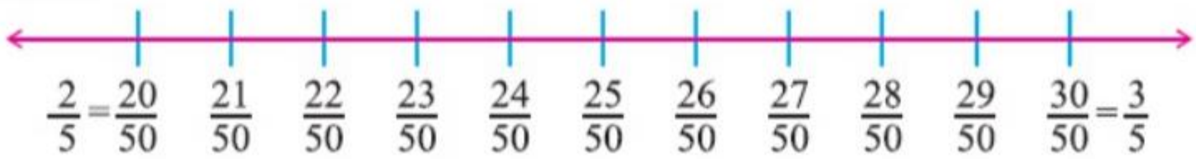


Rational numbers between two rational numbers :



Can you find more rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$?

Yes. We write $\frac{2}{5}$ as $\frac{20}{50}$ and $\frac{3}{5}$ as $\frac{30}{50}$, then we can find many rational numbers between them.

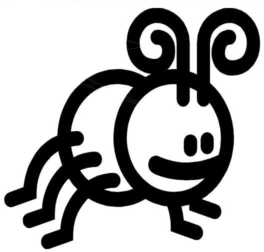


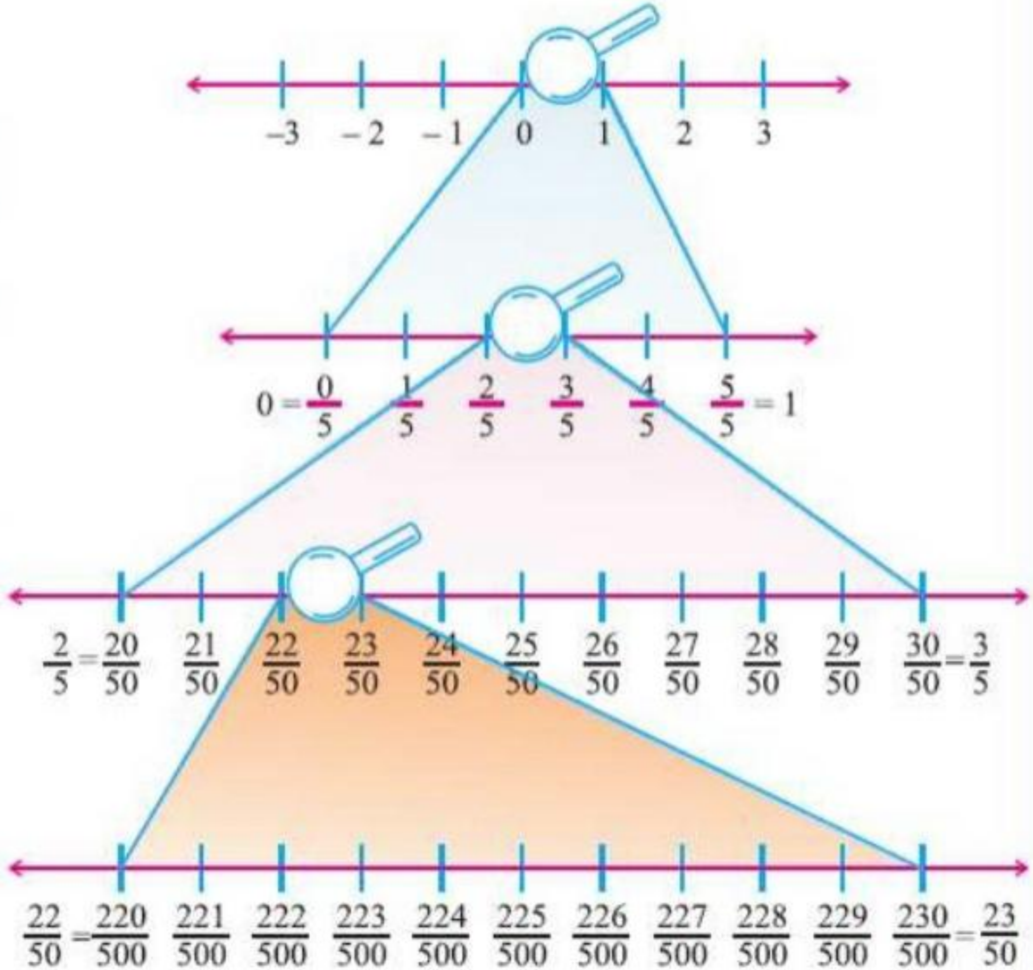
We can find nine rational numbers $\frac{21}{50}, \frac{22}{50}, \frac{23}{50}, \frac{24}{50}, \frac{25}{50}, \frac{26}{50}, \frac{27}{50}, \frac{28}{50}$ and $\frac{29}{50}$.

If we want to find some more rational numbers between $\frac{22}{50}$ and $\frac{23}{50}$, we write $\frac{22}{50}$ as $\frac{220}{500}$ and $\frac{23}{50}$ as $\frac{230}{500}$. Then we get nine rational numbers $\frac{221}{500}, \frac{222}{500}, \frac{223}{500}, \frac{224}{500}, \frac{225}{500}, \frac{226}{500}, \frac{227}{500}, \frac{228}{500}$ and $\frac{229}{500}$.

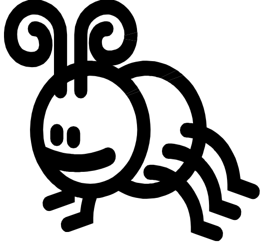
Let us understand this better with the help of the number line shown in the adjacent figure.

Observe the number line between 0 and 1 using a magnifying lens.





So, unlike natural numbers and integers, there are countless rational numbers between any two given rational numbers.



Rational numbers between two rational numbers :

We will find the rational numbers between given two rational numbers in two different ways :

- **LCM method**
- **Average method**

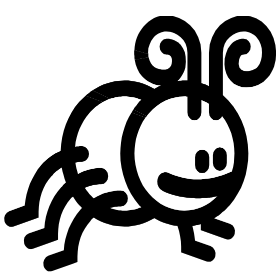
LCM METHOD

When you are given with two rational numbers and asked to find some rational numbers between them then follow the steps-

Let us consider two rational numbers $\frac{1}{2}$ and $\frac{1}{4}$ to find 3 rational numbers between them.

STEP 1 – Make sure that the denominator of the two fractions are same otherwise make the denominator same by taking the LCM of the denominators.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \quad \textit{and} \quad \frac{1}{4} \times \frac{1}{1} = \frac{1}{4}$$



LCM METHOD

STEP 2 – If the desired number of rational numbers are present in between the resultant rational numbers the answer is over.

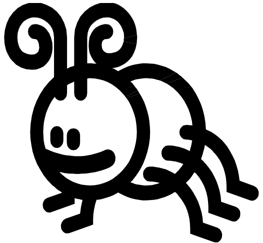
There are no rational numbers between $\frac{2}{4}$ *and* $\frac{1}{4}$.

STEP 3 – If desire number of rational numbers are not obtained then multiply suitable numbers at numerator as well as denominator and get the resultant rational numbers.

$$\frac{2}{4} \times \frac{5}{5} = \frac{10}{20} \quad \text{and} \quad \frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$$

So the rational numbers between $\frac{10}{20}$ *and* $\frac{5}{20}$ are :

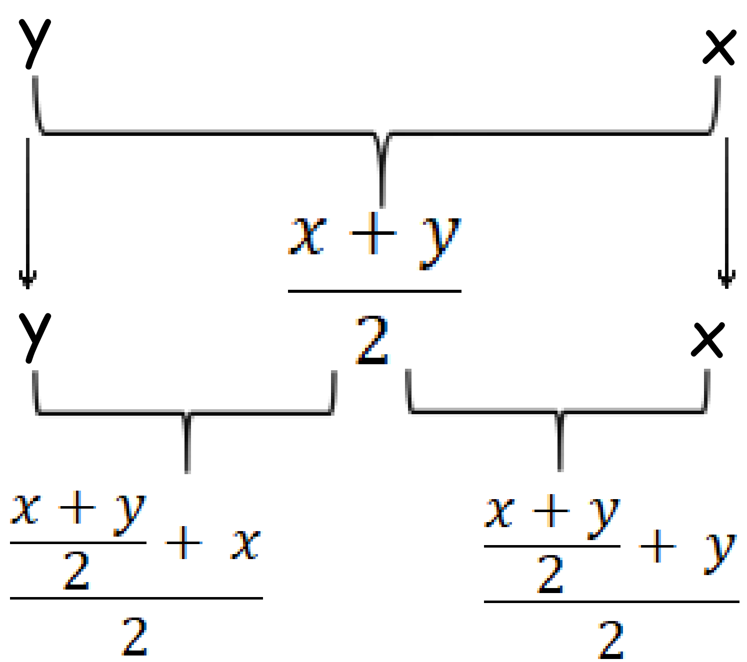
$$\frac{6}{20}, \frac{7}{20}, \frac{8}{20}, \frac{9}{20}$$

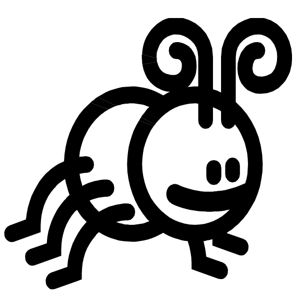


AVERAGE (MEAN) METHOD

Let us assume that 'x' and 'y' are two rational numbers.

- 1st rational number between x and y is $\frac{x+y}{2}$
- 2nd rational number between x and $\frac{x+y}{2}$ is $\frac{\frac{x+y}{2}+x}{2}$
- 3rd rational number between y and $\frac{x+y}{2}$ is $\frac{\frac{x+y}{2}+y}{2}$





AVERAGE (MEAN) METHOD

If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

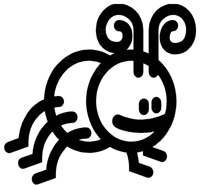
When you are given with two rational numbers and asked to find some rational numbers between them then follow the steps-

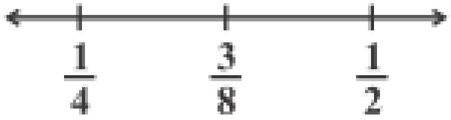
Let us consider two rational numbers $\frac{1}{2}$ and $\frac{1}{4}$ to find 3 rational numbers between them.

STEP 1 – Find average of $\frac{1}{2}$ and $\frac{1}{4}$.

$$\left(\frac{1}{4}+\frac{1}{2}\right) \div 2 = \left(\frac{1+2}{4}\right) \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

$$\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$$



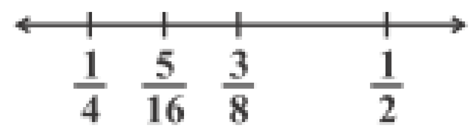


AVERAGE (MEAN) METHOD

STEP 2 – Find average of $\frac{1}{4}$ and $\frac{3}{8}$.

$$\left(\frac{1}{4}+\frac{3}{8}\right)\div 2=\frac{5}{8}\times\frac{1}{2}=\frac{5}{16}$$

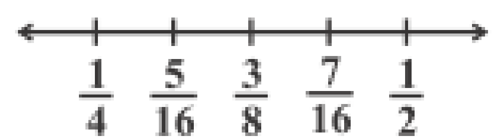
$$\frac{1}{4}<\frac{5}{16}<\frac{3}{8}<\frac{1}{2}$$



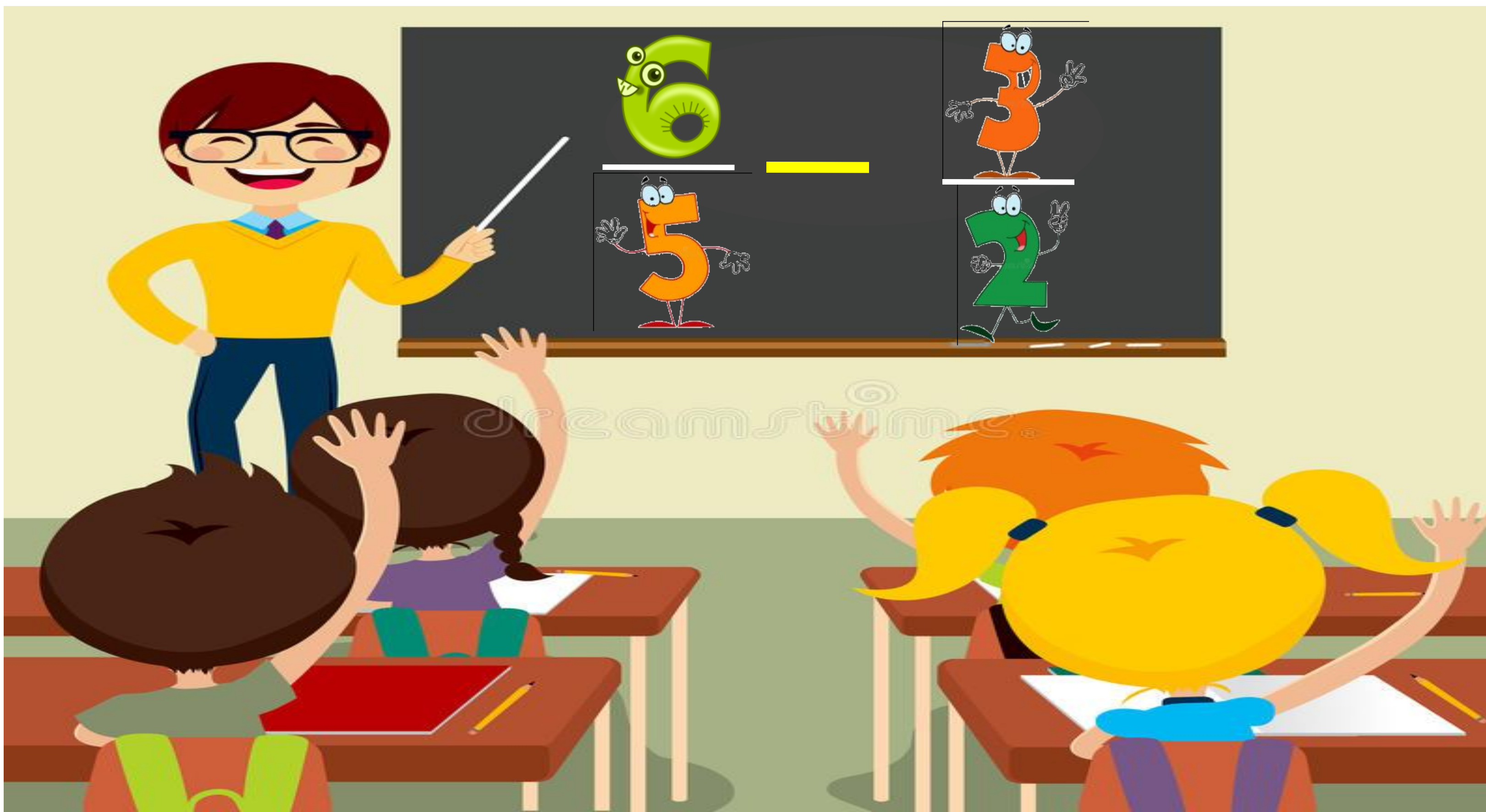
STEP 3 – Find average of $\frac{1}{2}$ and $\frac{3}{8}$.

$$\left(\frac{3}{8}+\frac{1}{2}\right)\div 2=\frac{7}{8}\times\frac{1}{2}=\frac{7}{16}$$

$$\frac{1}{4}<\frac{5}{16}<\frac{3}{8}<\frac{7}{16}<\frac{1}{2}$$



Let's summarize



COSURE PROPERTY

NUMBERS	CLOSED UNDER			
	ADDITION a+b is a rational number	SUBTRACTION a - b is a rational number	MULTIPLICATION a × b is a rational number	DIVISION a ÷ b is a rational number
RATIONAL NUMBERS	YES	YES	YES	YES

COMMUTATIVE PROPERTY

NUMBERS	COMMUTATIVE FOR			
	ADDITION a + b = b + a	SUBTRACTION a - b = b - a	MULTIPLICATION a × b = b × a	DIVISION a ÷ b = b ÷ a
RATIONAL NUMBERS	YES	NO	YES	NO

ASSOCIATIVE PROPERTY

NUMBERS	ASSOCIATIVE FOR			
	ADDITION (a + b) + c = a + (b + c)	SUBTRACTION (a - b) - c = a - (b - c)	MULTIPLICATION (a × b) × c = a × (b × c)	DIVISION (a ÷ b) ÷ c = a ÷ (b ÷ c)
RATIONAL NUMBERS	YES	NO	YES	NO

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. The **additive inverse** of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa.
6. The **reciprocal** or **multiplicative inverse** of the rational number $\frac{a}{b}$ is $\frac{c}{d}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.
7. **Distributivity** of rational numbers: For all rational numbers a, b and c,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

Think tank :

1) Which are the only two rational numbers that are reciprocal of their own?
2) Reciprocal of which number does not exist?
3) Which rational number is its own negative?